CHAPTER

1

ESSENTIALS OF GEOMETRY

For thousands of years, civilized people have used mathematics to investigate sizes, shapes, and the relationships among physical objects. Ancient Egyptians used geometry to solve many practical problems involving boundaries and land areas.

The work of Greek scholars such as Thales, Eratosthenes, Pythagoras, and Euclid for centuries provided the basis for the study of geometry in the Western world. About 300 B.C., Euclid and his followers organized the geometry of his day into a logical system contained in thirteen "books" known today as Euclid's *Elements*.

Euclid began his *Elements* with definitions:

- A *point* is that which has no part.
- A *line* is breathless length.
- The *extremities of a line* are points.
- A *straight line* is a line that lies evenly with the points on itself.
- A *surface* is that which has length and breadth only.

Today mathematicians consider the words *point*, *line*, and *straight line* to be undefined terms.

Euclid's *Elements* was, of course, written in Greek. One of the most commonly used translations is that of Sir Thomas L. Heath.

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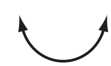
1-1 UNDEFINED TERMS

Geometry is the branch of mathematics that defines and relates the basic properties and measurement of line segments and angles. When we study mathematics, we reason about numbers and shapes. Most of the geometric and algebraic facts that are presented in this book you have already learned in previous mathematics courses. In this course we will investigate how these ideas interact to form a logical system. That logical system must start with generally accepted terms and concepts that are building blocks of the system.

There are four terms that we will accept without definition: *set*, *point*, *line*, and *plane*. These are called **undefined terms** because their meaning is accepted without definition. While we do not define these words, we will try to identify them in such a way that their meaning will be accepted by all.

A **set** is a collection of objects such that it is possible to determine whether a given object belongs to the collection or not. For example, the set of students in your mathematics class is determined by your teacher's class list. That list enables us to determine whether or not a student belongs to the set.

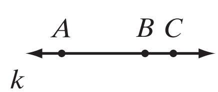
A **point** may be represented by a dot on a piece of paper, and is usually named by a capital letter. The dot shown, for example, represents a point, which we call point P. A geometric point has no length, width, or thickness; it only indicates place or position.


Curved Line

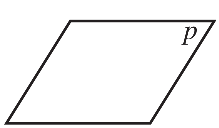
Straight Line

A **line** is a set of points. A set of points may form a curved line or a straight line as shown. Unless otherwise stated, the term *line* will mean a **straight line**.

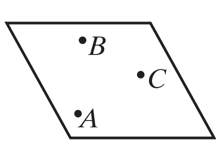
We understand what a straight line is by considering the set of points that can be arranged along a stretched string or along the edge of a ruler. While a stretched string or the edge of a ruler is limited in length, a line is an infinite set of points extending endlessly in both directions. Arrowheads are sometimes used to emphasize the fact that the line has no endpoints.



To name a line, we usually use two capital letters that name two points on the line. The two letters may be written in either order. The line shown here may be named line AB, written as $\overleftrightarrow{AB}$. It may also be named line AC or line BC, written as $\overleftrightarrow{AC}$ or $\overleftrightarrow{BC}$. The line may also be named by a single lowercase letter. Thus, the line on the left can also be called line k or simply k.



A **plane** is a set of points that form a flat surface extending indefinitely in all directions. A plane is suggested by the surface of the floor of a room or the surface of your desk. While a floor or your desk have boundaries, a plane extends endlessly in all directions. The figure here represents a plane, called plane p. This four-sided figure shows only part of a plane, however, because a plane has no boundaries.



A plane may also be named by using letters that identify three points in the plane that are not on the same line. The plane shown on the left is plane ABC.

Exercises

Writing About Mathematics

1. A section of the United States west of the Mississippi river is called the *Great Plains*. Explain why this area may be called a plane and how it is unlike a mathematical plane.

2. Explain why a stretched string is not a line.

Developing Skills

In 3–8, determine whether each statement is true or false.

3. A point has no length, width, or thickness.

4. A line is limited in length.

5. A plane has boundaries that are lines.

6. $\overleftrightarrow{AB}$ and $\overleftrightarrow{BA}$ name the same line.

7. A line is unlimited in length but has a definite thickness.

8. A point indicates place or position.

Applying Skills

9. Name three things that can be represented by a point.

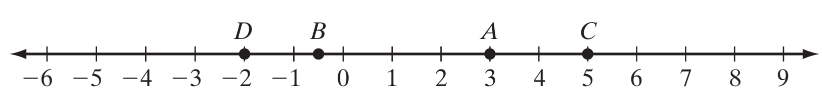
10. Name three things that can be represented by a line.

11. Name three things that can be represented by a plane.

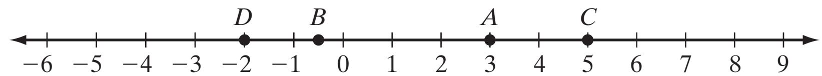
1-2 THE REAL NUMBERS AND THEIR PROPERTIES

As young children, our first mathematical venture was learning to count. As we continued to grow in our knowledge of mathematics, we expanded our knowledge of numbers to the set of integers, then to the set of rational numbers, and finally to the set of real numbers. We will use our understanding of the real numbers and their properties in our study of geometry.

In *Integrated Algebra 1*, we learned that every real number corresponds to a point on a **number line** and every point on the number line corresponds to a real number. The real number line below shows this correspondence for a part of the real numbers.



The number that corresponds to a point on a line is called the **coordinate** of the point. The coordinate of point A on the number line shown above is 3, and the coordinate of point B is $-\frac{1}{2}$. The point to which a number corresponds is called the **graph** of the number. The graph of 5 is C, and the graph of -2 is D.



Properties of the Real Number System

A **numerical operation** assigns a real number to every pair of real numbers. The following properties of the real numbers govern the operations of addition and multiplication.

1. *Closure*

The sum of two real numbers is a real number. This property is called the **closure property of addition**. For all real numbers a and b:

$$a + b \text{ is a real number.}$$

The product of two real numbers is a real number. This property is called the **closure property of multiplication**. For all real numbers a and b:

$$a \cdot b \text{ is a real number.}$$

2. *Commutative Property*

When we add real numbers, we can change the **order** in which two numbers are added without changing the sum. This property is called the **commutative property of addition**. For all real numbers a and b:

$$a + b = b + a$$

When we multiply real numbers, we can change the **order** of the factors without changing the product. This property is called the **commutative property of multiplication**. For all real numbers a and b:

$$a \cdot b = b \cdot a$$

3. *Associative Property*

When three numbers are added, two are added first and then their sum is added to the third. The sum does not depend on which two numbers are added first. This property is called the **associative property of addition**. For all real numbers a, b, and c:

$$(a + b) + c = a + (b + c)$$

When three numbers are multiplied, two are multiplied first and then their product is multiplied by the third. The product does not depend on which two numbers are multiplied first. This property is called the **associative property of multiplication**. For all real numbers a, b, and c:

$$a \times (b \times c) = (a \times b) \times c$$

4. *Identity Property*

When 0 is added to any real number a, the sum is a. The real number 0 is called the **additive identity**. For every real number a:

$$a + 0 = a \quad \text{and} \quad 0 + a = a$$

When 1 is multiplied by any real number a, the product is a. The real number 1 is called the **multiplicative identity**. For every real number a:

$$a \cdot 1 = a \quad \text{and} \quad 1 \cdot a = a$$

5. *Inverse Property*

Two real numbers are called **additive inverses** if their sum is the additive identity, 0. For every real number a, there exists a real number $-a$ such that:

$$a + (-a) = 0$$

Two real numbers are called **multiplicative inverses** if their product is the multiplicative identity, 1. For every real number $a \neq 0$, there exists a real number $\frac{1}{a}$ such that:

$$a \cdot \frac{1}{a} = 1$$

6. *Distributive Property*

The **distributive property** combines the operations of multiplication and addition. Multiplication distributes over addition. For all real numbers a, b, and c:

$$a(b + c) = ab + ac \quad \text{and} \quad (a + b)c = ac + bc$$

7. *Multiplication Property of Zero*

Zero has no multiplicative inverse. The **multiplication property of zero** defines multiplication by zero. For all real numbers a and b:

$$ab = 0 \text{ if and only if } a = 0 \text{ or } b = 0.$$

EXAMPLE 1

In **a–c**, name the property illustrated in each equality.

	Answers
a. $4(7 \cdot 5) = (4 \cdot 7)5$	Associative property of multiplication
b. $\sqrt{3} + 3 = 3 + \sqrt{3}$	Commutative property of addition
c. $\frac{1}{9}\left(4 + \sqrt{2}\right) = \frac{1}{9}(4) + \frac{1}{9}\left(\sqrt{2}\right)$	Distributive property of multiplication over addition

Exercises

Writing About Mathematics

1. a. Explain why the set of positive real numbers is not closed under subtraction.

b. Is the set of real numbers closed under subtraction? Explain your answer.

2. Explain why the set of negative real numbers is not closed under multiplication.

Developing Skills

3. Name the number that is the additive identity for the real numbers.

4. Name the number that is the multiplicative identity for the real numbers.

5. When $a = -11$, what does $-a$ represent?

6. Name the real number that has no multiplicative inverse.

In 7–16: **a.** Replace each question mark with a number that makes the statement true. **b.** Name the property illustrated in the equation that is formed when the replacement is made.

7. $7 + 14 = 14 + ?$

8. $7 + (?) = 0$

9. $3(4 \cdot 6) = (3 \cdot 4)?$

10. $11(9) = ?(11)$

11. $1x = ?$

12. $6(4 + b) = 6(4) + ?$

13. $17(?) = 1$

14. $\frac{1}{4}(24) = 24(?)$

15. $12(? + 10) = 12(10 + 4)$

16. $\pi\left(\frac{1}{\pi}\right) = ?$

17. For what value of x does $3x - 1$ have no multiplicative inverse?

18. If $ab = 0$ and $a \neq 0$, explain why $a + b = a$.

1-3 DEFINITIONS, LINES, AND LINE SEGMENTS

Qualities of a Good Definition

A **definition** is a statement of the meaning of a term. When we state a definition, we will use only undefined terms or terms that we have previously defined.

The definition of a term will state the set to which the term belongs and the qualities that makes it different from others in the set.

Collinear Points

DEFINITION

A **collinear set of points** is a set of points all of which lie on the same straight line.

This definition has the properties of a good definition:

1. It is expressed in undefined words.

   Set, *point*, and *line* are undefined words.

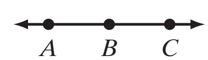
2. It states the set to which the defined term belongs.

   The set is a set of points.

3. It distinguishes the term from other members of the class.

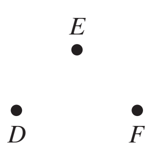
   The points must be on the same straight line.

A, *B*, and *C* are collinear points.



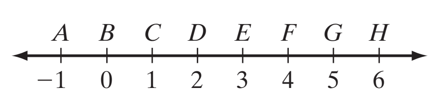
DEFINITION

A **noncollinear set of points** is a set of three or more points that do not all lie on the same straight line.



D, *E*, and *F* are noncollinear points.

The Distance Between Two Points



Every point on a line corresponds to a real number called its *coordinate*. To find the distance between any two points, we find the absolute value of the difference between the coordinates of the two points. For example, the distance between *G* and *C*, written as *GC*, is $|5 - 1| = 4$. We use absolute value so that the distance between any two points is independent of the order that we subtract the coordinates.

$$\begin{aligned} GC &= |5 - 1| \\ &= |4| \\ &= 4 \end{aligned} \qquad\qquad \begin{aligned} CG &= |1 - 5| \\ &= |-4| \\ &= 4 \end{aligned}$$

Recall that:

1. The absolute value of a positive number p is p. Thus, $|12| = 12$.

2. The absolute value of a negative number n is $-n$, the opposite of n. If $n = -7$, then $-n = -(-7) = 7$. Thus, $|-7| = 7$.

3. The absolute value of 0 is 0. Thus, $|0| = 0$.

DEFINITION

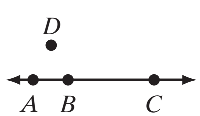
The **distance between two points on the real number line** is the absolute value of the difference of the coordinates of the two points.

On the real number line, the distance between point A, whose coordinate is a, and B, whose coordinate is b, can be represented as

$$AB = |a - b| = |b - a|.$$

Order of Points on a Line

When we learned to count, we placed the natural numbers in order, starting with the smallest. We count "one, two, three." We are saying that $1 < 2$ and $2 < 3$ or that $1 < 3$. When we locate the graphs of numbers on the number line, a number m is to the left of a number n if and only if $m < n$. In general, if the numbers m, n, and p occur in that order from left to right on the number line, then $m < n < p$. If $m < n$ and $n < p$, then n is between m and p on the number line.



In the figure, note that the position of point B with respect to A and C is different from the position of D with respect to A and C because A, B, and C are collinear but A, D, and C are noncollinear.

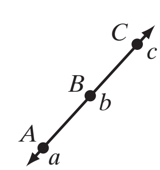
DEFINITION

B* is between *A* and *C if and only if A, B, and C are distinct collinear points and $AB + BC = AC$. This is also called **betweenness**.

The symbol $\overleftrightarrow{ABC}$ represents a line on which B is between A and C. Let a be the coordinate of A, b be the coordinate of B, and c be the coordinate of C.

In the figure, B is between A and C and $a < b < c$.

- Since $a < b$, $b - a$ is positive and $AB = |b - a| = b - a$.
- Since $b < c$, $c - b$ is positive and $BC = |c - b| = c - b$.
- Since $a < c$, $c - a$ is positive and $AC = |c - a| = c - a$.



Therefore,

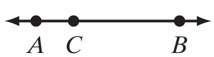
$$\begin{aligned} AB + BC &= (b - a) + (c - b) \\ &= c - a \\ &= AC \end{aligned}$$

Line Segments

A line segment is a subset, or a part of, a line. In the figure, A and B are two points of line m. Points A and B determine segment AB, written in symbols as $\overline{AB}$.

DEFINITION

A **line segment**, or a **segment**, is a set of points consisting of two points on a line, called endpoints, and all of the points on the line between the endpoints.



A line may be named by any two of the points on the line. A line segment is always named by its two endpoints. In the diagram, $\overleftrightarrow{AB}$, $\overleftrightarrow{AC}$, and $\overleftrightarrow{CB}$ all name the same line. However, $\overline{AB}$, $\overline{AC}$, and $\overline{CB}$ name different segments. To indicate that C is a point on $\overline{AB}$ but not one of its endpoints, we use the symbol $\overline{ACB}$.

DEFINITION

The **length** or **measure of a line segment** is the distance between its endpoints.

The length or measure of $\overline{AB}$ is written in symbols as AB. If the coordinate of A is a and the coordinate of B is b, then $AB = |b - a|$.

The line segments shown have the same length. These segments are said to be congruent.

DEFINITION

Congruent segments are segments that have the same measure.

The symbol $\cong$ is used to state that two segments are congruent. For example, since $\overline{AB}$ and $\overline{CD}$ are congruent, we write $\overline{AB} \cong \overline{CD}$. Because congruent segments have the same measure, when $\overline{AB} \cong \overline{CD}$, $AB = CD$. Note the correct symbolism. We write $\overline{AB} = \overline{CD}$ only when $\overline{AB}$ and $\overline{CD}$ are two different names for the same set of points.

In the figure, each segment, $\overline{AB}$ and $\overline{CD}$, is marked with two tick marks. When segments are marked with the same number of tick marks, the segments are known to be congruent. Therefore, $\overline{AB} \cong \overline{CD}$.

EXAMPLE 1

On a number line, the coordinate of A is -7 and the coordinate of B is 4. Find AB, the distance between A and B.

Solution The distance between A and B is the absolute value of the difference of their coordinates.

$$\begin{aligned} AB &= |(-7) - 4| \\ &= |-11| \\ &= 11 \quad \textit{Answer} \end{aligned}$$

EXAMPLE 2

R, T, and S are three points on a number line. The coordinate of R is 3, the coordinate of T is -7, and the coordinate of S is 9. Which point is between the other two?

Solution Since $-7 < 3 < 9$, R is between T and S. *Answer*

Note: We can state in symbols that R is between T and S by writing $\overline{TRS}$.

Exercises

Writing About Mathematics

1. Explain why "A hammer is a tool" is not a good definition.

2. Explain why "A hammer is used to drive nails" is not a good definition.

Developing Skills

In 3–7: **a.** Write the definition that each phrase describes. **b.** State the class to which each defined term belongs. **c.** Tell what distinguishes each defined term from other members of the class.

3. noncollinear set of points

4. distance between any two points on the number line

5. line segment

6. measure of a line segment

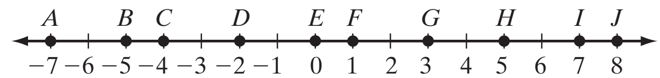
7. congruent segments

In 8–13, use the number line to find each measure.

8. AB **9.** BD

10. CD **11.** FH

12. GJ **13.** EJ



14. Name three segments on the number line shown above that are congruent.

15. Three segments have the following measures: $PQ = 12$, $QR = 10$, $PR = 18$. What conclusion can you draw about P, Q, and R? Draw a diagram to justify your answer.

Applying Skills

16. A carpenter is measuring the board with which he is working.

a. Because one end of the measuring tape is unreadable, the carpenter places the measuring tape so that one end of the board is at 3 inches and the other at 15 inches. How long is the board?

b. The carpenter wants to cut the board so that he has a piece 10 inches long. If he leaves the measuring tape in the position described in **a**, at what point on the tape should he mark the board to make the cut? (Two answers are possible.)

17. A salesperson drove from Troy to Albany, a distance of 8 miles, and then from Albany to Schenectady, a distance of 16 miles. The distance from Schenectady back to Troy is 18 miles. Are Troy, Albany, and Schenectady in a straight line? Explain your answer.

1-4 MIDPOINTS AND BISECTORS

Midpoint of a Line Segment

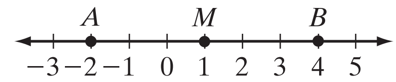
A line segment can be divided into any number of congruent parts. The point that divides the line segment into two equal parts is called the **midpoint** of the segment.

DEFINITION

The **midpoint of a line segment** is a point of that line segment that divides the segment into two congruent segments.

If M is the midpoint of $\overline{AB}$, then $\overline{AM} \cong \overline{MB}$ or $AM = MB$. It is also true that $AM = \frac{1}{2}AB$, that $MB = \frac{1}{2}AB$, that $AB = 2AM$, and that $AB = 2MB$.

On the number line, the coordinate of A is -2 and the coordinate of B is 4. Therefore,



$$\begin{aligned} AB &= |-2 - 4| \\ &= |-6| \\ &= 6 \end{aligned}$$

If M is the midpoint of $\overline{AB}$, then $AM = 3$ and $MB = 3$. The coordinate of M can be found by adding 3 to the smaller coordinate, -2. The coordinate of M is $-2 + 3 = 1$. The coordinate of M can also be found by subtracting 3 from the larger coordinate, 4. The coordinate of M is $4 - 3 = 1$.

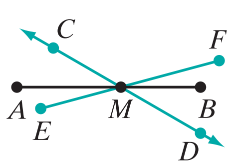
This example suggests the following relationship for finding the midpoint of a segment:

► **For any line segment $\overline{AB}$, if the coordinate of A is a and the coordinate of B is b, then the coordinate of the midpoint of $\overline{AB}$ is $\frac{a + b}{2}$.**

Bisector of a Line Segment

DEFINITION

The **bisector of a line segment** is any line, or subset of a line, that intersects the segment at its midpoint.



The line segment $\overline{AB}$ is bisected at M if M is the midpoint of $\overline{AB}$. Any line such as $\overleftrightarrow{CD}$, any segment such as $\overline{EF}$, and any ray such as $\overrightarrow{MC}$ are bisectors of $\overline{AB}$ because they intersect $\overline{AB}$ at its midpoint M.

Note: A line, unlike a line segment, cannot be bisected because it extends indefinitely in two directions.

Adding and Subtracting Line Segments

DEFINITION

A line segment, $\overline{RS}$ is the **sum of two line segments**, $\overline{RP}$ and $\overline{PS}$, if P is between R and S.

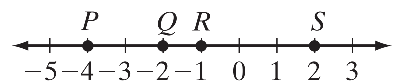
For three collinear points R, P, and S, with P between R and S, $\overline{RP} + \overline{PS}$ indicates the line segment formed by R, P, and S, that is, $\overline{RPS}$. Then $\overline{RP} + \overline{PS}$ is the *union* of segments $\overline{RP}$ and $\overline{PS}$. The expression $\overline{RP} + \overline{PS}$ represents a line segment only when P is between R and S.

We can also refer to the *lengths* of the line segments. Thus, $RS = RP + PS$. Recall that if P is between R and S, then R, P, and S are collinear. Observe that P divides $\overline{RS}$ into two segments, $\overline{RP}$ and $\overline{PS}$. The following equalities are true for the lengths of the segments.

$$RS = RP + PS \qquad RP = RS - PS \qquad PS = RS - RP$$

EXAMPLE 1

The points P, Q, R, and S are shown as points on a number line.



a. Find PQ, PR, QR, RS, PR, QS, and PS.

b. What point is the midpoint of $\overline{PS}$?

c. Show that $PQ + QS = PS$.

Solution **a.** $PQ = |-2 - (-4)| = |-2 + 4| = 2$

$PR = |-1 - (-4)| = |-1 + 4| = 3$

$QR = |-1 - (-2)| = |-1 + 2| = 1$

$RS = |2 - (-1)| = |2 + 1| = 3$

$QS = |2 - (-2)| = |2 + 2| = 4$

$PS = |2 - (-4)| = |2 + 4| = 6$

b. Since $PR = RS$ and $PR + RS = PS$, R is the midpoint of $\overline{PS}$.

c. $PQ + QS = 2 + 4 = 6$ and $PS = 6$. Therefore, $PQ + QS = PS$.

Exercises

Writing About Mathematics

1. Is it ever correct to write $\overleftrightarrow{AB} = \overleftrightarrow{CD}$? If so, what does that imply about A, B, C, and D?

2. If $AM = MB$, does this necessarily mean that M is the midpoint of $\overline{AB}$? Explain why or why not.

Developing Skills

In 3–6, state two conclusions that can be drawn from the given data. In one conclusion, use the symbol $\cong$; in the other, use the symbol $=$.

3. T is the midpoint of $\overline{AC}$.

4. $\overline{MN}$ bisects $\overline{RS}$ at N.

5. C is the midpoint of $\overline{BD}$.

6. $\overleftrightarrow{LM}$ bisects $\overline{ST}$ at P.

7. A, B, C, D are points on $\overleftrightarrow{AD}$, B is between A and C, C is between B and D, and $\overline{AB} \cong \overline{CD}$.

a. Draw $\overleftrightarrow{AD}$ showing B and C.

b. Explain why $\overline{AC} \cong \overline{BD}$.

8. What is the coordinate of the midpoint of $\overline{DE}$ if the coordinate of D is -4 and the coordinate of E is 9?

9. S, M, and T are points on the number line. The coordinate of M is 2, and the coordinate of T is 14. What is the coordinate of S if M is the midpoint of $\overline{ST}$?

10. P, Q, and R are points on the number line. The coordinate of P is -10, the coordinate of Q is 6, and the coordinate of R is 8.

a. Does $PQ + QR = PR$?

b. Is Q the midpoint of $\overline{PR}$? If not, what is the coordinate of the midpoint?

Applying Skills

11. Along the New York State Thruway, there are distance markers that show the distance in miles from the beginning of the Thruway. Alicia is at marker 215 and will exit the Thruway at marker 395. She wants to stop to rest at the midpoint of her trip. At what distance marker should she stop?

12. A carpenter is measuring the board with which she is working.

a. The carpenter places a measuring tape so that one end of the board is at 24 inches and the other at 8 inches. How long is the board?

b. The carpenter wants to cut the board so that she has two pieces of equal length. If she leaves the measuring tape in the position described in **a**, at what point on the tape should she mark the board to make the cut?

Hands-On Activity

For this activity, use a sheet of wax paper, patty paper, or other paper thin enough to see through. You may work alone or with a partner.

1. Draw a line segment $\overline{AB}$.

2. Fold the paper so that A coincides with B. Crease the paper. Label the crease $\overline{CDE}$ with D the point at which $\overline{CDE}$ intersects $\overline{AB}$.

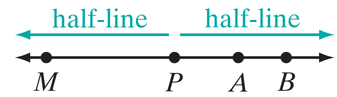
3. What is true about $\overline{AB}$ and D? $\overline{AD}$ and $\overline{DB}$?

1-5 RAYS AND ANGLES

Half-Lines and Rays

DEFINITION

Two points, A and B, are **on one side of a point** P if A, B, and P are collinear and P is not between A and B.

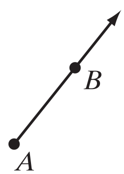


Every point on a line divides the line into two opposite sets of points called half-lines. A **half-line** consists of the set of all points on one side of the point of division. The point of division itself does not belong to the half-line. In the figure, point P divides $\overleftrightarrow{MB}$ into two half-lines.

DEFINITION

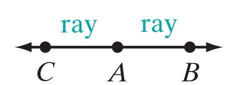
A **ray** consists of a point on a line and all points on one side of the point.

This definition tells us that a ray is a set of all the points in a half-line and the dividing point. The dividing point is called the **endpoint** of the ray.



A ray is named by placing an arrow that points to the right over two capital letters. The first letter must name the endpoint of the ray. The second letter may be the name of any other point on the ray. The figure shows ray AB, which starts at endpoint A, contains point B, and extends indefinitely in one direction. Ray AB is written as $\overrightarrow{AB}$.

Note in the figure that the ray whose endpoint is O may be named $\overrightarrow{OA}$ or $\overrightarrow{OB}$. However, it may not be named $\overrightarrow{AB}$ since the first capital letter in the name of a ray must represent its endpoint.



A point on a line creates two different rays, each with the same endpoint. In the figure, $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are opposite rays because points A, B, and C are collinear and points B and C are not on the same side of point A, but are on opposite sides of point A.

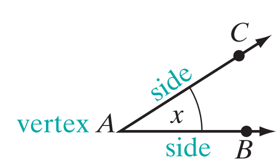
DEFINITION

Opposite rays are two rays of the same line with a common endpoint and no other point in common.

Angles

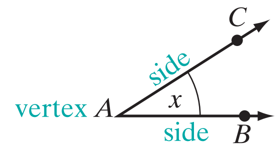
DEFINITION

An **angle** is a set of points that is the union of two rays having the same endpoint.



In the figure, $\overrightarrow{AB}$ and $\overrightarrow{AC}$, which form an angle, are called the **sides** of the angle. The endpoint of each ray, A, is called the **vertex** of the angle. The symbol for angle is $\angle$.

An angle, such as the one illustrated, may be named in any of the following ways:



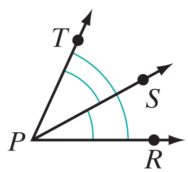
1. By a capital letter that names its vertex.

   Example: $\angle A$

2. By a lowercase letter or number placed inside the angle.

   Example: $\angle x$

3. By three capital letters, the middle letter naming the vertex and each of the other letters naming a point on a different ray. Example: $\angle BAC$ or $\angle CAB$

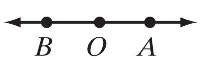


When a diagram shows several angles with the same vertex, we avoid confusion by using three letters to name each angle. For example, in the figure, the smaller angles are $\angle RPS$ and $\angle SPT$; the larger angle is $\angle RPT$.

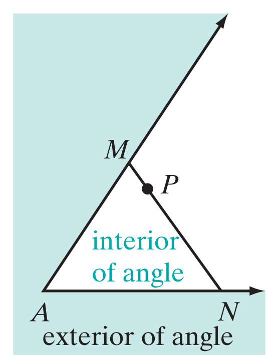
Straight Angles

DEFINITION

A **straight angle** is an angle that is the union of opposite rays.



The sides of a straight angle belong to the same straight line. In the figure, $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are opposite rays of $\overleftrightarrow{AB}$ because they have a common endpoint, O, and no other points in common. Thus, $\angle AOB$ is a straight angle because it is the union of the opposite rays $\overrightarrow{OA}$ and $\overrightarrow{OB}$.



An angle divides the points of a plane that are not points of the angle into two sets or regions. One region is called the **interior of the angle**; the other region is called the **exterior of the angle**. In the figure, $\angle A$ is not a straight angle. M is any point on one side of $\angle A$ and N is any point on the other side of the angle.

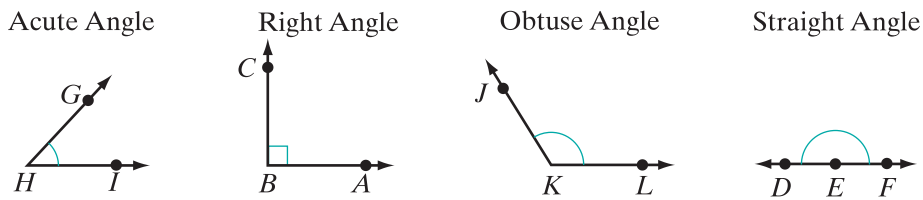
Neither M nor N is the vertex of the angle. Any point P on $\overline{MN}$ between points M and N belongs to the interior region of the angle. The region consisting of all points such as P is called the *interior of the angle*. All other points of the plane, except the points of the angle itself, form the region called the *exterior of the angle*.

The Measure of an Angle

To measure an angle, we must select a unit of measure and know how this unit is to be applied. In geometry, the unit used to measure an angle is usually a *degree*. The number of degrees in an angle is called its *degree measure*.

The degree measure of a straight angle is 180. A **degree** is, therefore, the measure of an angle that is $\frac{1}{180}$ of a straight angle. It is possible to use units other than the degree to measure angles. In this book, when we write $m\angle HRC = 45$, we mean that the measure of $\angle HRC$ is 45 degrees.

Classifying Angles According to Their Measures



DEFINITION

An **acute angle** is an angle whose degree measure is greater than 0 and less than 90.

In the figure above, $\angle GHI$ is an acute angle.

DEFINITION

A **right angle** is an angle whose degree measure is 90.

In the figure above, $\angle ABC$ is a right angle. Therefore, we know that $m\angle ABC = 90$. The symbol ⏋ at B is used to show that $\angle ABC$ is a right angle.

DEFINITION

An **obtuse angle** is an angle whose degree measure is greater than 90 and less than 180.

In the figure above, $\angle JKL$ is an obtuse angle.

A **straight angle**, previously defined as the union of opposite rays, is an angle whose degree measure is 180. In the figure above, DEF is a straight angle. Thus, $m\angle DEF = 180$. Note that $\overrightarrow{ED}$ and $\overrightarrow{EF}$, the sides of $\angle DEF$, are opposite rays and form a straight line.

EXAMPLE 1

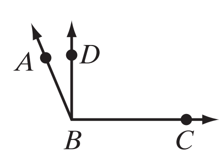
D is a point in the interior of $\angle ABC$, $m\angle ABD = 15$, and $m\angle DBC = 90$.

a. Find $m\angle ABC$.

b. Name an acute angle.

c. Name a right angle.

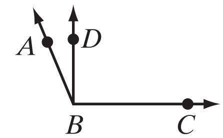
d. Name an obtuse angle.



Solution **a.** $m\angle ABC = m\angle ABD + m\angle DBC$

$= 15 + 90$

$= 105$



b. $\angle ABD$ is an acute angle.

c. $\angle DBC$ is a right angle.

d. $\angle ABC$ is an obtuse angle.

Answers **a.** $m\angle ABC = 105$ **b.** $\angle ABD$ **c.** $\angle DBC$ **d.** $\angle ABC$

Exercises

Writing About Mathematics

1. Explain the difference between a half-line and a ray.

2. Point R is between points P and S. Are $\overrightarrow{PR}$ and $\overrightarrow{PS}$ the same ray, opposite rays, or neither the same nor opposite rays? Explain your answer.

Developing Skills

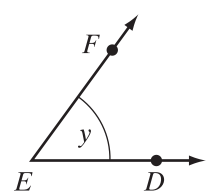
3. For the figure shown:

a. Name two rays.

b. Name the vertex of the angle.

c. Name the sides of the angle.

d. Name the angle in four ways.



4. a. Name the vertex of $\angle BAD$ in the figure.

b. Name the sides of $\angle BAD$.

c. Name all the angles with A as vertex.

d. Name the angle whose sides are $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

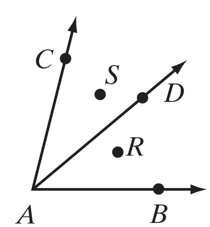
e. Name the ray that is a side of both $\angle BAD$ and $\angle BAC$.

f. Name two angles in whose interior regions point R lies.

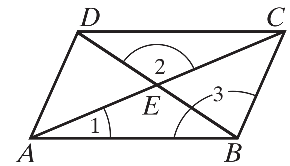
g. Name the angle in whose exterior region point S lies.

h. Are $\overrightarrow{AB}$ and $\overrightarrow{AC}$ opposite rays? Explain your answer.

i. Is $\angle BAC$ a straight angle? Explain your answer.



5. For the figure shown:



a. Name angle 1 in four other ways.

b. Name angle 2 in two other ways.

c. Name angle 3 in two other ways.

d. Name the point of intersection of $\overline{AC}$ and $\overline{BD}$.

e. Name two pairs of opposite rays.

f. Name two straight angles each of which has its vertex at E.

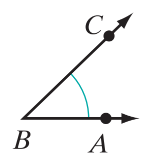
g. Name two angles whose sum is $\angle ABC$.

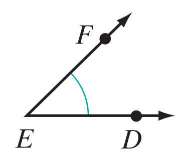
1-6 MORE ANGLE DEFINITIONS

Congruent Angles

DEFINITION

Congruent angles are angles that have the same measure.





In the figures, $\angle ABC$ and $\angle DEF$ have the same measure. This is written as $m\angle ABC = m\angle DEF$. Since congruent angles are angles that have the same measure, we may also say that $\angle ABC$ and $\angle DEF$ are congruent angles, symbolized as $\angle ABC \cong \angle DEF$. We may use either notation.

$\angle ABC \cong \angle DEF$	The angles are congruent.
$m\angle ABC = m\angle DEF$	The measures of the angles are equal.

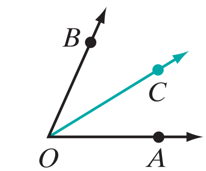
Note: We do not write $\angle ABC = \angle DEF$. Two angles are equal only if they name the union of the same rays. For example, $\angle ABC = \angle CBA$.

When two angles are known to be congruent, we may mark each one with the same number of arcs, as shown in the figures above, where each angle is marked with a single arc.

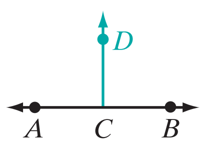
Bisector of an Angle

DEFINITION

A **bisector of an angle** is a ray whose endpoint is the vertex of the angle, and that divides that angle into two congruent angles.



For example, if $\overrightarrow{OC}$ is the bisector of $\angle AOB$, then $\angle AOC \cong \angle COB$ and $m\angle AOC = m\angle COB$. If $\overrightarrow{OC}$ bisects $\angle AOB$, we may say that $m\angle AOC = \frac{1}{2}m\angle AOB$, $m\angle COB = \frac{1}{2}m\angle AOB$, $m\angle AOB = 2m\angle AOC$, and $m\angle AOB = 2m\angle COB$.



Let C be any point on $\overleftrightarrow{AB}$, $\angle ACB$ is a straight angle, and $m\angle ACB = 180$. If $\overrightarrow{CD}$ is the bisector of $\angle ACB$, each of the two congruent angles, $\angle ACD$ and $\angle DCB$, has degree measure 90 and is a right angle. Any line, ray, or line segment that is a subset of $\overleftrightarrow{CD}$, and contains C, is said to be perpendicular to $\overleftrightarrow{AB}$ at C. For example, $\overline{CD}$ is perpendicular to $\overleftrightarrow{AB}$.

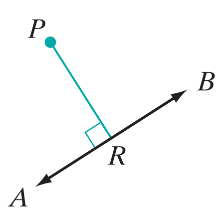
DEFINITION

Perpendicular lines are two lines that intersect to form right angles.

Since rays and line segments are contained in lines, any rays or line segments that intersect to form right angles are also perpendicular.

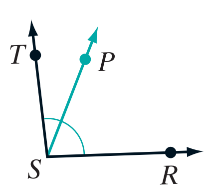
DEFINITION

The **distance from a point to a line** is the length of the perpendicular from the point to the line.



P is a point not on $\overleftrightarrow{AB}$, and $\overline{PR} \perp \overleftrightarrow{AB}$. The segment $\overline{PR}$ is called the perpendicular from P to $\overleftrightarrow{AB}$. The point R at which the perpendicular meets the line is called the **foot** of the perpendicular. The distance from P to $\overleftrightarrow{AB}$ is PR (the length of $\overline{PR}$).

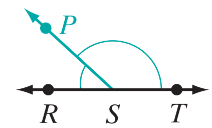
Adding and Subtracting Angles



DEFINITION

If point P is a point in the interior of $\angle RST$ and $\angle RST$ is not a straight angle, or if P is any point not on straight angle RST, then $\angle RST$ is the **sum of two angles**, $\angle RSP$ and $\angle PST$.

In the figure, $\overrightarrow{SP}$ separates $\angle RST$ into two angles, $\angle RSP$ and $\angle PST$, such that the following relations are true for the measures of the angles:



$$m\angle RST = m\angle RSP + m\angle PST$$
$$m\angle RSP = m\angle RST - m\angle PST$$
$$m\angle PST = m\angle RST - m\angle RSP$$

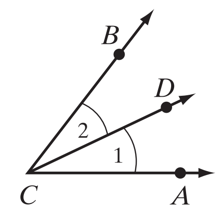
EXAMPLE 1

In the figure, $\overrightarrow{CD}$ bisects $\angle ACB$.

a. Name $\angle 1$ in two other ways.

b. Write a conclusion that states that two angles are congruent.

c. Write a conclusion that states that two angles have equal measures.



Solution **a.** $\angle ACD$ and $\angle DCA$. *Answer*

b. $\angle ACD \cong \angle BCD$ or $\angle 2 \cong \angle 1$. *Answer*

c. $m\angle ACD = m\angle BCD$ or $m\angle 2 = m\angle 1$. *Answer*

Exercises

Writing About Mathematics

1. Tanya said that if $\overleftrightarrow{RST}$ separates $\angle PSQ$ into two congruent angles, then $\overleftrightarrow{RST}$ is the bisector of $\angle PSQ$. Do you agree with Tanya? Explain why or why not.

2. If an obtuse angle is bisected, what kind of an angle is each of the two angles formed?

Developing Skills

In 3–10, if an angle contains the given number of degrees, **a.** state whether the angle is an acute angle, a right angle, an obtuse angle, or a straight angle, **b.** find the measure of each of the angles formed by the bisector of the given angle.

3. 24 **4.** 98 **5.** 126 **6.** 90

7. 82 **8.** 180 **9.** 57 **10.** 3

In 11–13, find the measure of each of the following:

11. $\frac{1}{2}$ of a right angle **12.** $\frac{1}{3}$ of a right angle **13.** $\frac{4}{5}$ of a straight angle

In 14 and 15, in each case, use the information to:

a. Draw a diagram.

b. Write a conclusion that states the congruence of two angles.

c. Write a conclusion that states the equality of the measures of two angles.

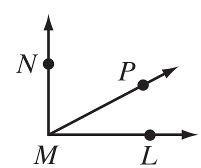
14. $\overrightarrow{CD}$ bisects $\angle ACB$.

15. $\overrightarrow{AC}$ is the bisector of $\angle DAB$.

16. If a straight angle is bisected, what types of angles are formed?

17. If a right angle is bisected, what types of angles are formed?

In 18–20, complete each statement, which refers to the figure shown.

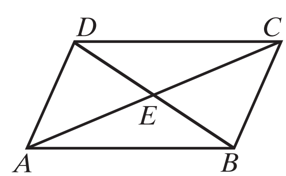


18. $m\angle LMN = m\angle LMP + m\angle$______

19. $m\angle LMP = m\angle LMN - m\angle$______

20. $m\angle LMN - m\angle$______ $= m\angle NMP$

In 21–24, use the figure to answer each question.



21. $m\angle ABE + m\angle EBC = m\angle$______

22. $m\angle BEC + m\angle CED = m\angle$______

23. $m\angle ADC - m\angle CDE = m\angle$______

24. $m\angle AEC - m\angle AEB = m\angle$______

25. Every horizontal line is perpendicular to every vertical line. Draw, using pencil and paper or geometry software, a horizontal line *ACB* and a vertical line *DCF*.

a. Name four congruent angles in your diagram.

b. What is the measure of each of the angles that you named in part **a**?

c. Name two rays that bisect $\angle ACB$.

d. Name two rays that bisect $\angle DCF$.

Applying Skills

26. $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ are opposite rays and $\overrightarrow{PS}$ bisects $\angle QPR$. If $m\angle QPS$ is represented by $4x + 30$, what is the value of x?

27. $\overrightarrow{BD}$ bisects $\angle ABC$. If $m\angle ABD$ can be represented by $3a + 10$ and $m\angle DBC$ can be represented by $5a - 6$, what is $m\angle ABC$?

28. $\overrightarrow{RT}$ bisects $\angle QRS$. If $m\angle QRS = 10x$ and $m\angle SRT = 3x + 30$, what is $m\angle QRS$?

29. $\overrightarrow{BD}$ bisects $\angle ABC$ and $\overrightarrow{MP}$ bisects $\angle LMN$. If $m\angle CBD = m\angle PMN$, is $\angle ABC \cong \angle LMN$? Justify your answer.

Hands-On Activity

For this activity, use a sheet of wax paper, patty paper, or other paper thin enough to see through. You may work alone or with a partner.

1. Draw a straight angle, $\angle ABC$.

2. Fold the paper through the vertex of the angle so that the two opposite rays correspond, and crease the paper. Label two points, D and E, on the crease with B between D and E.

3. Measure each angle formed by the crease and one of the opposite rays that form the straight angle. What is true about these angles?

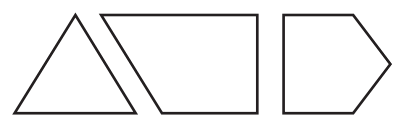
4. What is true about $\overline{DE}$ and $\overline{AC}$?

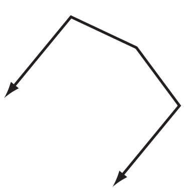
1-7 TRIANGLES

DEFINITION

A **polygon** is a closed figure in a plane that is the union of line segments such that the segments intersect only at their endpoints and no segments sharing a common endpoint are collinear.

A polygon consists of three or more line segments, each of which is a **side** of the polygon. For example, a triangle has three sides, a quadrilateral has four sides, a pentagon has five sides, and so on.





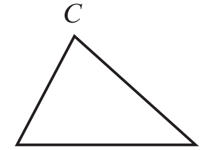
In the definition of a polygon, we used the word *closed*. We understand by the word *closed* that, if we start at any point on the figure and trace along the sides, we will arrive back at the starting point. In the diagram, we see a figure that is not closed and is, therefore, not a polygon.

DEFINITION

A **triangle** is a polygon that has exactly three sides.

The polygon shown is triangle ABC, written as $\triangle ABC$. In $\triangle ABC$, each of the points A, B, and C is a vertex of the triangle. $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$ are the sides of the triangle.

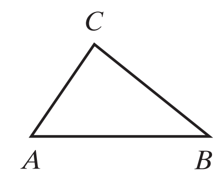
C



A B

Included Sides and Included Angles of a Triangle

If a line segment is the side of a triangle, the endpoints of that segment are the vertices of two angles. For example, in $\triangle ABC$, the endpoints of $\overline{AB}$ are the vertices of $\angle A$ and $\angle B$. We say that the side, $\overline{AB}$, is **included** between the angles, $\angle A$ and $\angle B$. In $\triangle ABC$:



1. $\overline{AB}$ is included between $\angle A$ and $\angle B$.

2. $\overline{BC}$ is included between $\angle B$ and $\angle C$.

3. $\overline{CA}$ is included between $\angle C$ and $\angle A$.

In a similar way, two sides of a triangle are subsets of the rays of an angle, and we say that the angle is **included** between those sides. In $\triangle ABC$:

1. $\angle A$ is included between $\overline{AC}$ and $\overline{AB}$.

2. $\angle B$ is included between $\overline{BA}$ and $\overline{BC}$.

3. $\angle C$ is included between $\overline{CA}$ and $\overline{CB}$.

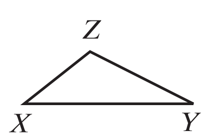
Opposite Sides and Opposite Angles in a Triangle

For each side of a triangle, there is one vertex of the triangle that is not an endpoint of that side. For example, in $\triangle ABC$, C is not an endpoint of $\overline{AB}$. We say that $\overline{AB}$ is the side **opposite** $\angle C$ and that $\angle C$ is the angle **opposite** side $\overline{AB}$. Similarly, $\overline{AC}$ is opposite $\angle B$ and $\angle B$ is opposite $\overline{AC}$; also $\overline{BC}$ is opposite $\angle A$ and $\angle A$ is opposite $\overline{BC}$.

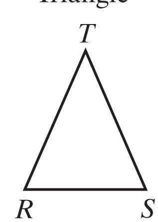
The length of a side of a triangle may be represented by the lowercase form of the letter naming the opposite vertex. For example, in $\triangle ABC$, $BC = a$, $CA = b$, and $AB = c$.

Classifying Triangles According to Sides

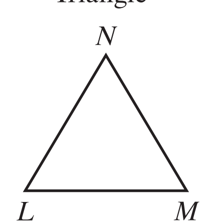
Scalene Triangle



Isosceles Triangle



Equilateral Triangle



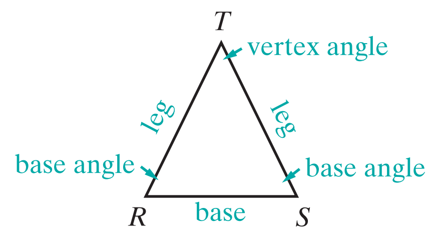
DEFINITION

A **scalene triangle** is a triangle that has no congruent sides.

An **isosceles triangle** is a triangle that has two congruent sides.

An **equilateral triangle** is a triangle that has three congruent sides.

Parts of an Isosceles Triangle

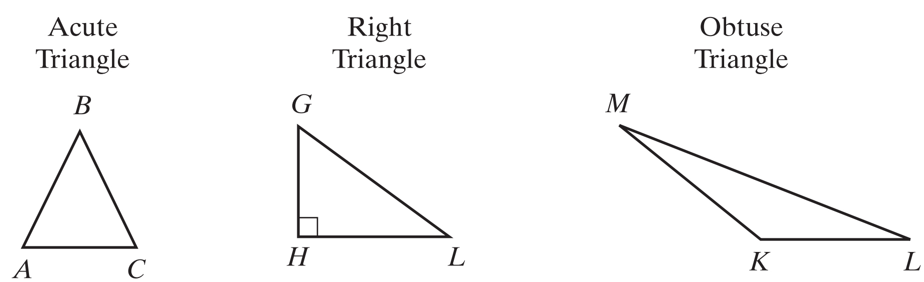


In isosceles triangle RST, the two congruent sides, $\overline{TR}$ and $\overline{TS}$, are called the **legs** of the triangle. The third side, $\overline{RS}$, is called the **base**.

The angle formed by the two congruent sides of the isosceles triangle, $\angle T$, is called the **vertex angle** of the isosceles triangle. The vertex angle is the angle opposite the base.

The angles whose vertices are the endpoints of the base of the triangle, $\angle R$ and $\angle S$, are called the **base angles** of the isosceles triangle. The base angles are opposite the legs.

Classifying Triangles According to Angles



DEFINITION

An **acute triangle** is a triangle that has three acute angles.

A **right triangle** is a triangle that has a right angle.

An **obtuse triangle** is a triangle that has an obtuse angle.

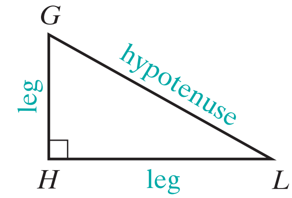
In each of these triangles, two of the angles may be congruent. In an acute triangle, all of the angles may be congruent.

DEFINITION

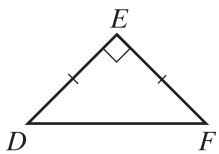
An **equiangular triangle** is a triangle that has three congruent angles.

Right Triangles

In right triangle GHL, the two sides of the triangle that form the right angle, $\overline{GH}$ and $\overline{HL}$, are called the **legs** of the right triangle. The third side of the triangle, $\overline{GL}$, the side opposite the right angle, is called the **hypotenuse**.



EXAMPLE 1



In $\triangle DEF$, m$\angle E$ = 90 and $EF = ED$.

	Answers
a. Classify the triangle according to sides.	The triangle is an isosceles triangle.
b. Classify the triangle according to angles.	The triangle is a right triangle.
c. What sides are the legs of the triangle?	The legs of the triangle are $\overline{EF}$ and $\overline{ED}$.
d. What side is opposite the right angle?	$\overline{FD}$ is opposite the right angle.
e. What angle is opposite $\overline{EF}$?	Angle D is opposite $\overline{EF}$.
f. What angle is included between $\overline{EF}$ and $\overline{FD}$?	Angle F is between $\overline{EF}$ and $\overline{FD}$.

Using Diagrams in Geometry

In geometry, diagrams are used to display and relate information. Diagrams are composed of line segments that are parallel or that intersect to form angles.

From a diagram, we may assume that:

- A line segment is a part of a straight line.
- The point at which two segments intersect is a point on each segment.
- Points on a line segment are between the endpoints of that segment.
- Points on a line are collinear.
- A ray in the interior of an angle with its endpoint at the vertex of the angle separates the angle into two adjacent angles.

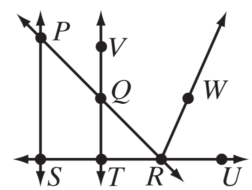
From a diagram, we may NOT assume that:

- The measure of one segment is greater than, or is equal to, or is less than that of another segment.
- A point is a midpoint of a line segment.

- The measure of one angle is greater than, is equal to, or is less than that of another angle.
- Lines are perpendicular or that angles are right angles (unless indicated with an angle bracket).
- A given triangle is isosceles or equilateral (unless indicated with tick marks).
- A given quadrilateral is a parallelogram, rectangle, square, rhombus, or trapezoid.

EXAMPLE 2

Tell whether or not each statement may be assumed from the given diagram.



(1) $\overleftrightarrow{PR}$ is a straight line. Yes

(2) S, T, and R are collinear. Yes

(3) $\angle PSR$ is a right angle. No

(4) $\triangle QTR$ is isosceles. No

(5) $\angle QRW$ is adjacent to $\angle WRU$. Yes

(6) Q is between V and T. Yes

(7) $\overline{PQR}$ and $\overline{VQT}$ intersect. Yes

(8) R is the midpoint of $\overline{TU}$. No

(9) $QT < PS$ No

(10) $m\angle QRW > m\angle VQP$ No

(11) $PQTS$ is a trapezoid. No

Exercises

Writing About Mathematics

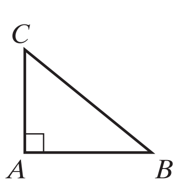
1. Is the statement "A triangle consists of three line segments" a good definition? Justify your answer.

2. Explain the difference between the legs of an isosceles triangle and the legs of a right triangle.

Developing Skills

In 3 and 4, name the legs and the hypotenuse in each right triangle shown.

3.



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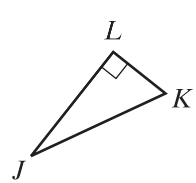
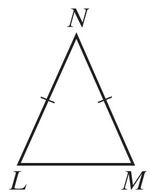
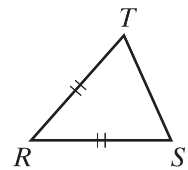


In 5 and 6, name the legs, the base, the vertex angle, and the base angles in each isosceles triangle shown.

5.



6.



In 7–10, sketch, using pencil and paper or geometry software, each of the following. Mark congruent sides and right angles with the appropriate symbols.

7. An acute triangle that is isosceles

8. An obtuse triangle that is isosceles

9. A right triangle that is isosceles

10. An obtuse triangle that is scalene

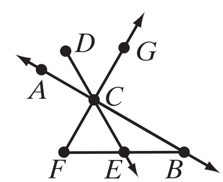
11. The vertex angle of an isosceles triangle is $\angle ABC$. Name the base angles of this triangle.

12. In $\triangle DEF$, $\overline{DE}$ is included between which two angles?

13. In $\triangle RST$, $\angle S$ is included between which two sides?

14. **a.** Name three things that can be assumed from the diagram to the right.

b. Name three things that can *not* be assumed from the diagram to the right.



Applying Skills

15. The degree measures of the acute angles of an obtuse triangle can be represented by $5a + 12$ and $3a - 2$. If the sum of the measures of the acute angles is 82, find the measure of each of the acute angles.

16. The measures of the sides of an equilateral triangle are represented by $x + 12$, $3x - 8$, and $2x + 2$. What is the measure of each side of the triangle?

17. The lengths of the sides of an isosceles triangle are represented by $2x + 5$, $2x - 6$, and $3x - 3$. What are the lengths of the sides of the triangle? (*Hint:* the length of a line segment is a positive quantity.)

18. The measures of the sides of an isosceles triangle are represented by $x + 5$, $3x + 13$, and $4x + 11$. What are the measures of each side of the triangle? Two answers are possible.

CHAPTER SUMMARY

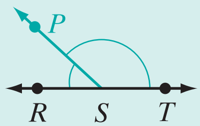
Undefined Terms

- set, point, line, plane

Definitions to Know

- A **collinear set of points** is a set of points all of which lie on the same straight line.
- A **noncollinear set of points** is a set of three or more points that do not all lie on the same straight line.
- The **distance between two points on the real number line** is the absolute value of the difference of the coordinates of the two points.
- ***B* is between *A* and *C*** if and only if A, B, and C are distinct collinear points and $AB + BC = AC$.
- A **line segment**, or a **segment**, is a set of points consisting of two points on a line, called endpoints, and all of the points on the line between the endpoints.
- The **length** or **measure of a line segment** is the distance between its endpoints.
- **Congruent segments** are segments that have the same measure.
- The **midpoint of a line segment** is a point of that line segment that divides the segment into two congruent segments.
- The **bisector of a line segment** is any line, or subset of a line, that intersects the segment at its midpoint.
- A line segment, $\overline{RS}$, is the **sum of two line segments**, $\overline{RP}$ and $\overline{PS}$, if P is between R and S.
- Two points, A and B, are **on one side of a point** P if A, B, and P are collinear and P is not between A and B.
- A **half-line** is a set of points on one side of a point.
- A **ray** is a part of a line that consists of a point on the line, called an endpoint, and all the points on one side of the endpoint.
- **Opposite rays** are two rays of the same line with a common endpoint and no other point in common.
- An **angle** is a set of points that is the union of two rays having the same endpoint.
- A **straight angle** is an angle that is the union of opposite rays and whose degree measure is 180.
- An **acute angle** is an angle whose degree measure is greater than 0 and less than 90.
- A **right angle** is an angle whose degree measure is 90.
- An **obtuse angle** is an angle whose degree measure is greater than 90 and less than 180.

- **Congruent angles** are angles that have the same measure.
- A **bisector of an angle** is a ray whose endpoint is the vertex of the angle, and that divides that angle into two congruent angles.
- **Perpendicular lines** are two lines that intersect to form right angles.
- The **distance from a point to a line** is the length of the perpendicular from the point to the line.



- If point P is a point in the interior of $\angle RST$ and $\angle RST$ is not a straight angle, or if P is any point not on straight angle RST, then $\angle RST$ is the **sum of two angles**, $\angle RSP$ and $\angle PST$.
- A **polygon** is a closed figure in a plane that is the union of line segments such that the segments intersect only at their endpoints and no segments sharing a common endpoint are collinear.
- A **triangle** is a polygon that has exactly three sides.
- A **scalene triangle** is a triangle that has no congruent sides.
- An **isosceles triangle** is a triangle that has two congruent sides.
- An **equilateral triangle** is a triangle that has three congruent sides.
- An **acute triangle** is a triangle that has three acute angles.
- A **right triangle** is a triangle that has a right angle.
- An **obtuse triangle** is a triangle that has an obtuse angle.
- An **equiangular triangle** is a triangle that has three congruent angles.

Properties of the Real Number System

	Addition	Multiplication
Closure	$a + b$ is a real number.	$a \cdot b$ is a real number.
Commutative Property	$a + b = b + a$	$a \cdot b = b \cdot a$
Associative Property	$(a + b) + c = a + (b + c)$	$a \times (b \times c) = (a \times b) \times c$
Identity Property	$a + 0 = a$ and $0 + a = a$	$a \cdot 1 = a$ and $1 \cdot a = a$
Inverse Property	$a + (-a) = 0$	$a \neq 0, a \cdot \frac{1}{a} = 1$
Distributive Property	$a(b + c) = ab + ac$ and $ab + ac = a(b + c)$	
Multiplication Property of Zero	$ab = 0$ if and only if $a = 0$ or $b = 0$	

VOCABULARY

1-1 Undefined term • Set • Point • Line • Straight line • Plane

1-2 Number line • Coordinate • Graph • Numerical operation • Closure property of addition • Closure property of multiplication • Commutative property of addition • Commutative property of multiplication • Associative property of addition • Associative property of multiplication • Additive identity • Multiplicative identity • Additive inverses • Multiplicative inverses • Distributive property • Multiplication property of zero •

1-3 Definition • Collinear set of points • Noncollinear set of points • Distance between two points on the real number line • Betweenness • Line segment • Segment • Length (Measure) of a line segment • Congruent segments

1-4 Midpoint of a line segment • Bisector of a line segment • Sum of two line segments

1-5 Half-line • Ray • Endpoint • Opposite rays • Angle • Sides of an angle • Vertex • Straight angle • Interior of an angle • Exterior of an angle • Degree • Acute angle • Right angle • Obtuse angle • Straight angle

1-6 Congruent angles • Bisector of an angle • Perpendicular lines • Distance from a point to a line • Foot • Sum of two angles

1-7 Polygon • Side • Triangle • Included side • Included angle • Opposite side • Opposite angle • Scalene triangle • Isosceles triangle • Equilateral triangle • Legs • Base • Vertex angle • Base angles • Acute triangle • Right triangle • Obtuse triangle • Equiangular triangle • Hypotenuse

REVIEW EXERCISES

1. Name four undefined terms.

2. Explain why “A line is like the edge of a ruler” is not a good definition.

In 3–10, write the term that is being defined.

3. The set of points all of which lie on a line.

4. The absolute value of the difference of the coordinates of two points.

5. A polygon that has exactly three sides.

6. Any line or subset of a line that intersects a line segment at its midpoint.

7. Two rays of the same line with a common endpoint and no other points in common.

8. Angles that have the same measure.

9. A triangle that has two congruent sides.

10. A set of points consisting of two points on a line and all points on the line between these two points.

11. In right triangle LMN, $m\angle M = 90$. Which side of the triangle is the hypotenuse?

12. In isosceles triangle RST, $RS = ST$.

a. Which side of the triangle is the base?

b. Which angle is the vertex angle?

13. Points D, E, and F lie on a line. The coordinate of D is -3, the coordinate of E is 1, and the coordinate of F is 9.

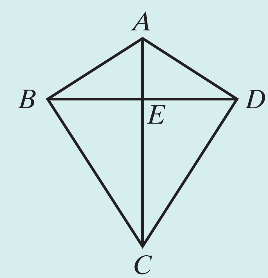
a. Find DE, EF, and DF.

b. Find the coordinate of M, the midpoint of $\overline{DF}$.

c. $\overleftrightarrow{AB}$ intersects $\overline{EF}$ at C. If $\overleftrightarrow{AB}$ is a bisector of $\overline{EF}$, what is the coordinate of C?

14. Explain why a line segment has only one midpoint but can have many bisectors.

In 15–18, use the figure shown. In polygon $ABCD$, $\overline{AC}$ and $\overline{BD}$ intersect at E, the midpoint of $\overline{BD}$.



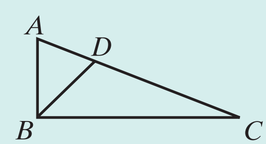
15. Name two straight angles.

16. What angle is the sum of $\angle ADE$ and $\angle EDC$?

17. Name two congruent segments.

18. What line segment is the sum of $\overline{AE}$ and $\overline{EC}$?

In 19–22, use the figure shown. $\overline{AB} \perp \overline{BC}$ and $\overrightarrow{BD}$ bisects $\angle ABC$.



19. Name two congruent angles.

20. What is the measure of $\angle ABD$?

21. Name a pair of angles the sum of whose measures is 180.

22. Does $AB + BC = AC$? Justify your answer.

Exploration

Euclidean geometry, which we have been studying in this chapter, focuses on the plane. **Non-Euclidian geometry** focuses on surfaces other than the plane. For instance, *spherical geometry* focuses on the sphere. In this Exploration, you will draw on a spherical object, such as a grapefruit or a Styrofoam ball, to relate terms from Euclidean geometry to spherical geometry.

1. Draw a point on your sphere. How does this point compare to a point in Euclidean geometry?
2. The shortest distance between two points is called a **geodesic**. In Euclidean geometry, a line is a geodesic. Draw a second point on your sphere. Connect the points with a geodesic and extend it as far as possible. How does this geodesic compare to a line in Euclidean geometry?
3. Draw a second pair of points and a second geodesic joining them on your sphere. The intersection of the geodesics forms angles. How do these angles compare to an angle in Euclidean geometry?
4. Draw a third pair of points and a third geodesic joining them on your sphere. This will form triangles, which by definition are polygons with exactly three sides. How does a triangle on a sphere compare to a triangle in Euclidean geometry? Consider both sides and angles.